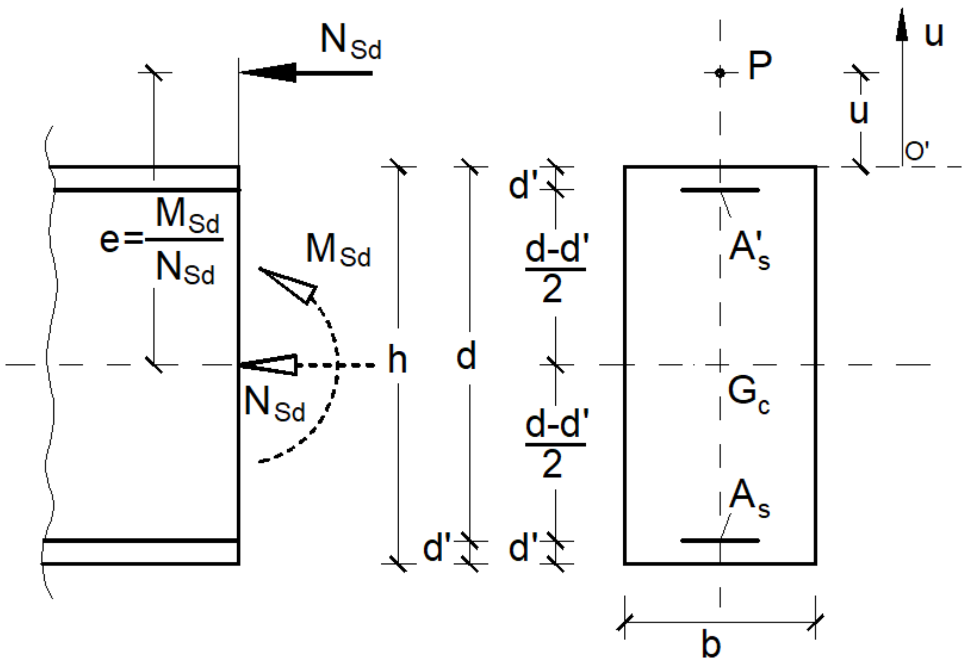
Flessione composta



- **Equazione di equilibrio alla traslazione**

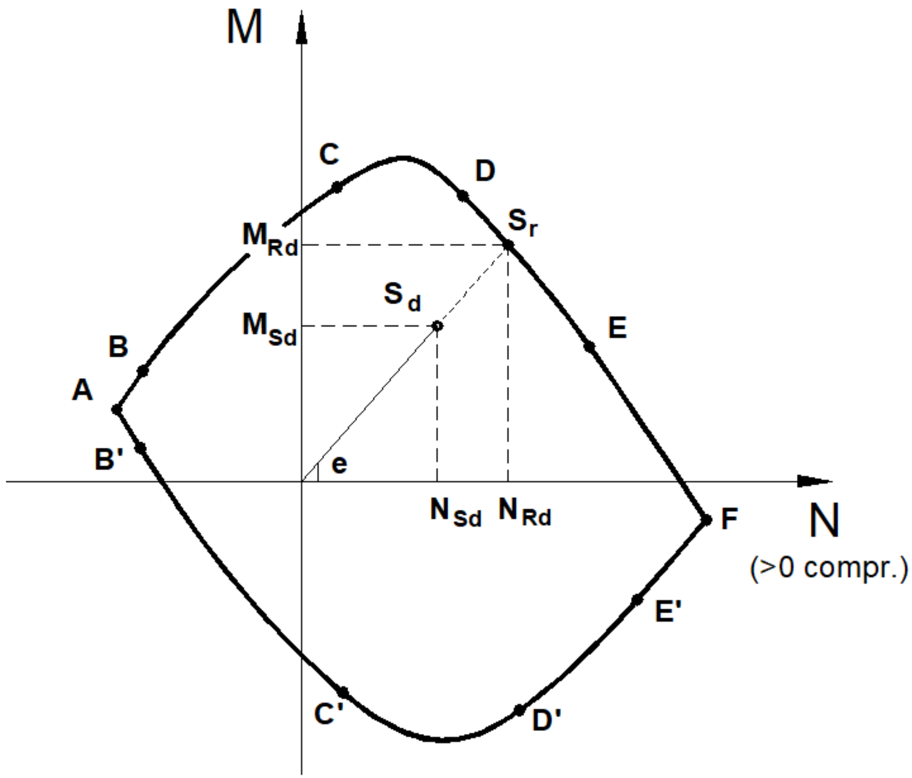
$$F'_s + F_c - F_s = N_{sd}$$

- **Equazione di equilibrio alla rotazione intorno a P**

$$F'_s(d' + u) + F_c(u + x'_c) - F_s(d + u) = 0$$

Stati Limite Ultimi per Tensioni normali

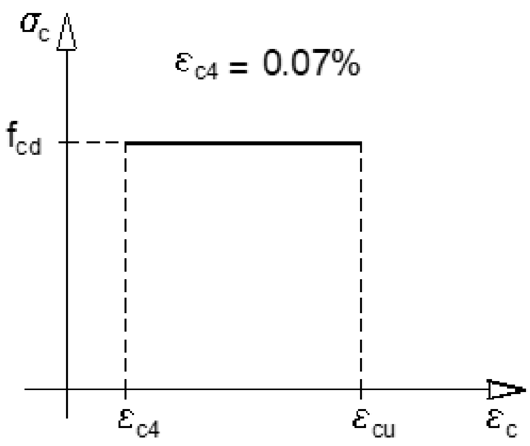
Flessione composta: Domini di interazione



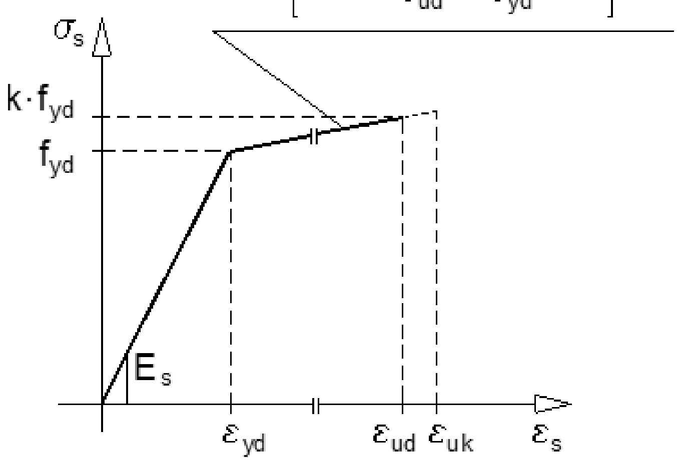
La frontiera di rottura è una curva chiusa e convessa. Essa dipende dalle caratteristiche geometriche della sezione di calcestruzzo, dalla posizione e dal quantitativo di armatura in essa presente, dalle caratteristiche meccaniche del cls e dell'acciaio.

La costruzione del dominio di interazione di una sezione si effettua considerando tutte le possibili rette di rottura e scrivendo:

- **Equazione di equilibrio alla traslazione nella direzione dell'asse della elemento**
- **Equazione di equilibrio alla rotazione intorno all'asse baricentrico della sezione geometrica**



$$\sigma_s = \left[(k-1)\,\frac{\varepsilon_s - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1\right] f_{yd}$$



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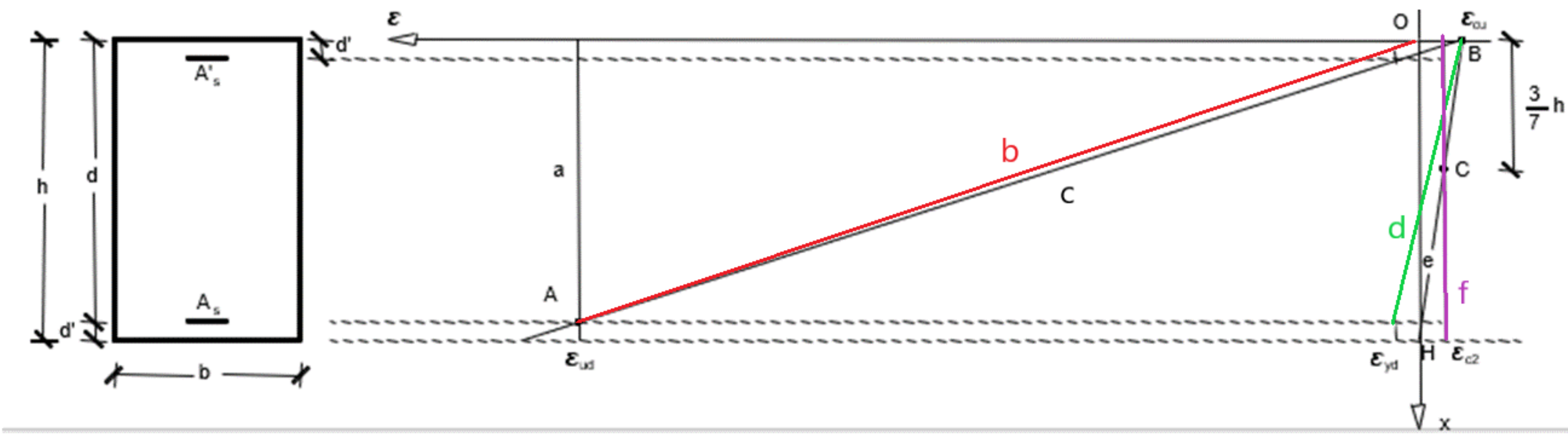
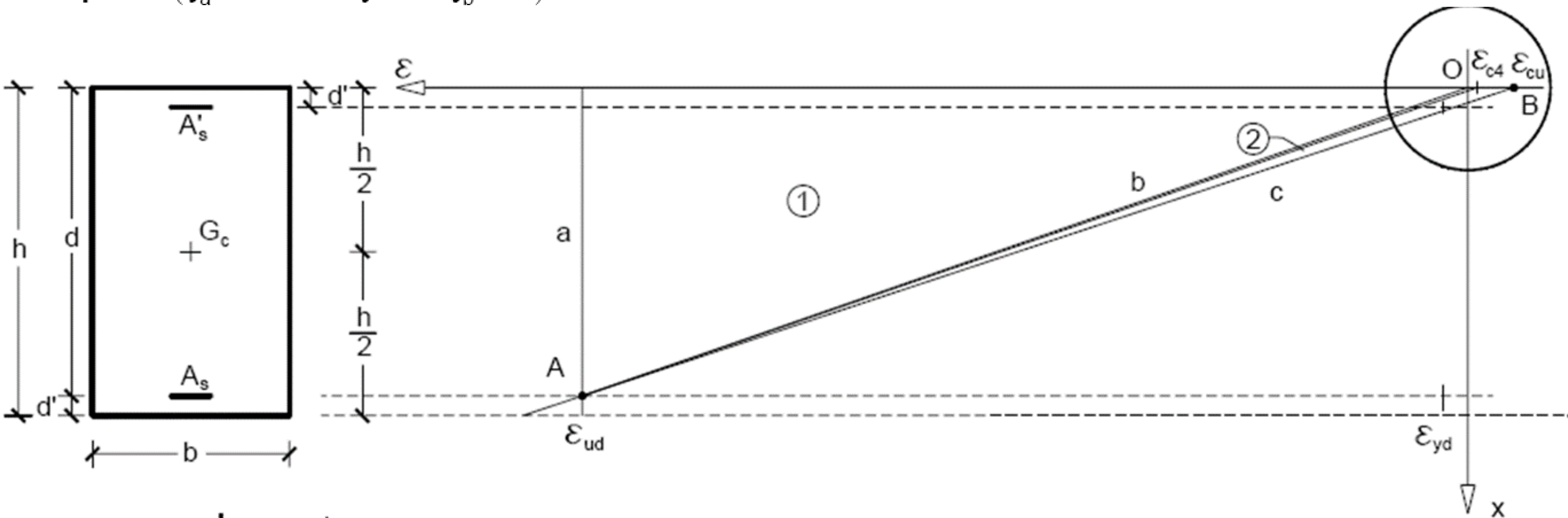
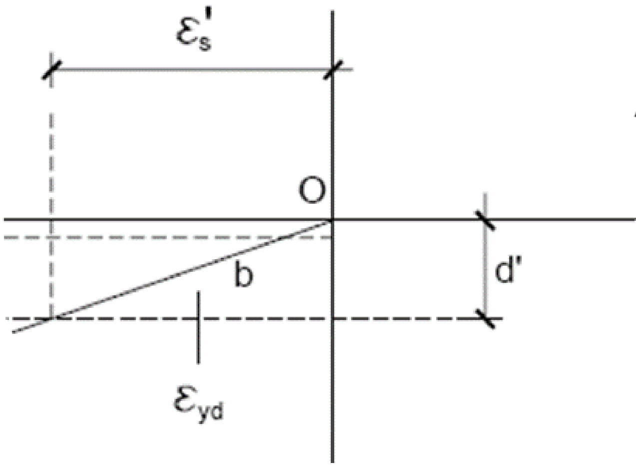
Domini di interazione



Campo 1 $\left(\xi_a = -\infty < \xi \leq \xi_b = 0\right)$





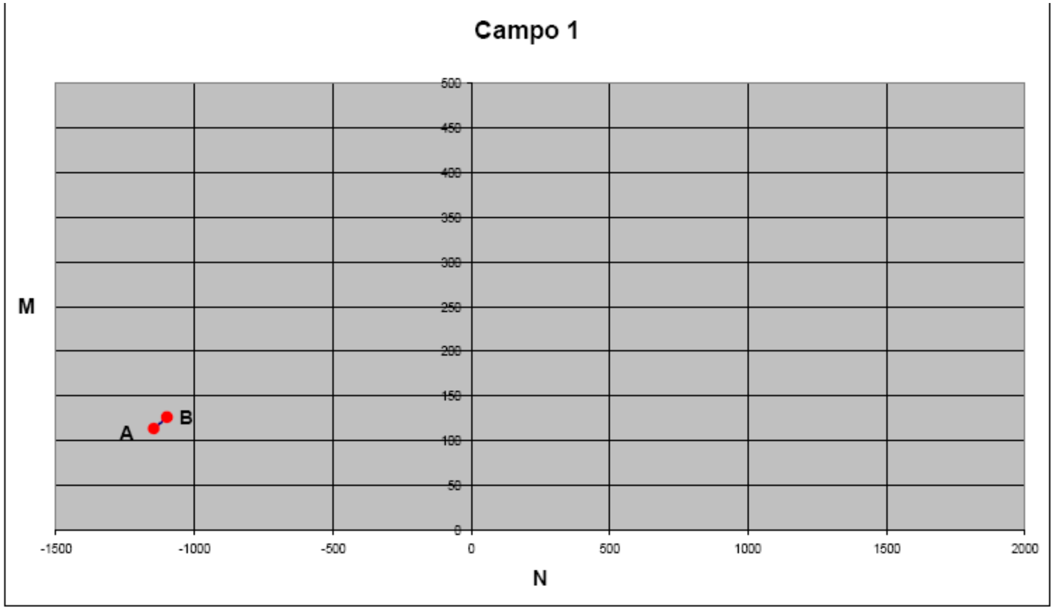
Assumendo $\delta \geq 0.06$ (valore usuale nelle realizzazioni pratiche),

$$\varepsilon'_s \geq 0.06 \cdot 6.75\% = 0.405\% > \varepsilon_{yd} = 0.196\% ;$$

Armatura superiore snervata in campo 1

Campo 1 $\left(\xi_a = -\infty < \xi \leq \xi_b = 0\right)$

$$\begin{cases} N_a = -k\left(A_s + A'_s\right)f_{yd} \\ M_a = kf_{yd}\left(A_s - A'_s\right)\dfrac{d-d'}{2} \end{cases} \quad ; \quad \begin{cases} N_b = -kf_{yd}A_s - \sigma'_s A'_s \\ M_b = \left(kf_{yd}A_s - \sigma'_s A'_s\right)\dfrac{d-d'}{2} \end{cases}$$

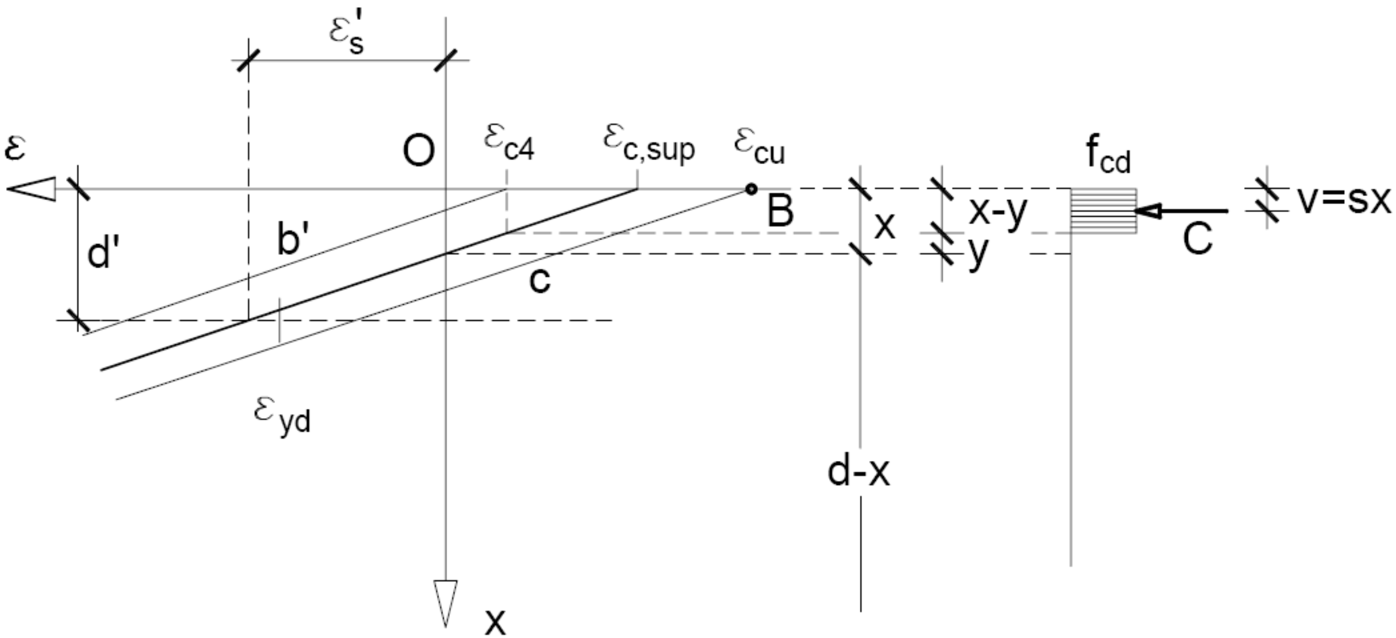


dove

$$\sigma'_s = \left[(k-1)\frac{\varepsilon'_s - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1\right] f_{yd}$$

Se k=1 → A ≡ B

Campo 2 $\left(\xi_b = 0 \le \xi \le \xi_c = 0.049\right)$



$$\xi^* = \frac{x^*}{d} = \frac{\varepsilon_{c4}}{\varepsilon_{ud} + \varepsilon_{c4}} = \frac{0.07\%}{6.75\% + 0.07\%} = 0.01026$$

Posizione dell'inizio dello stress block

$$\frac{y}{\varepsilon_{c4}} = \frac{x}{\varepsilon_{c,sup}} \quad \Rightarrow \quad y = \frac{\varepsilon_{c4}}{\varepsilon_{c,sup}} x = \frac{\varepsilon_{c4}}{\varepsilon_{ud}}(d-x) \quad \Rightarrow \quad x - y = x - \frac{\varepsilon_{c4}}{\varepsilon_{ud}}(d-x) = \left[1 - \frac{\varepsilon_{c4}}{\varepsilon_{ud}} \cdot \frac{d-x}{x}\right] x = \left[1 - \frac{\varepsilon_{c4}}{\varepsilon_{ud}} \cdot \frac{1-\xi}{\xi}\right] x = \alpha(\xi) \cdot x$$

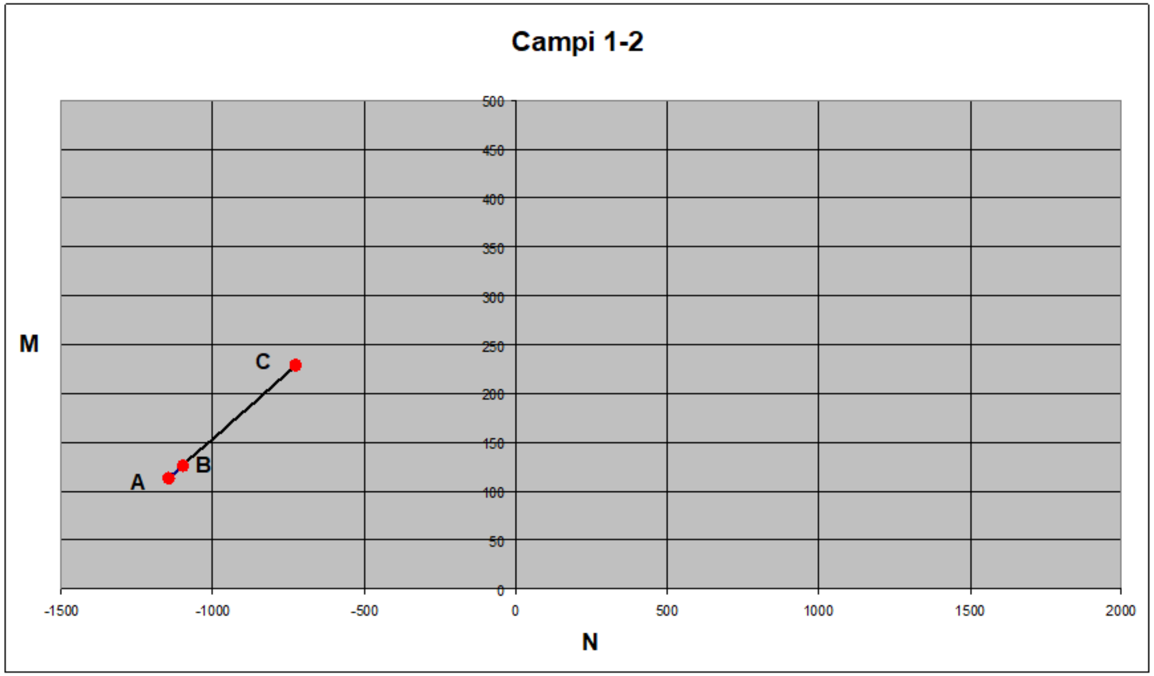
ξ	≤ 0.01026	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.049
α	0.00	0.32	0.49	0.60	0.66	0.71	0.75	0.78	0.80

,

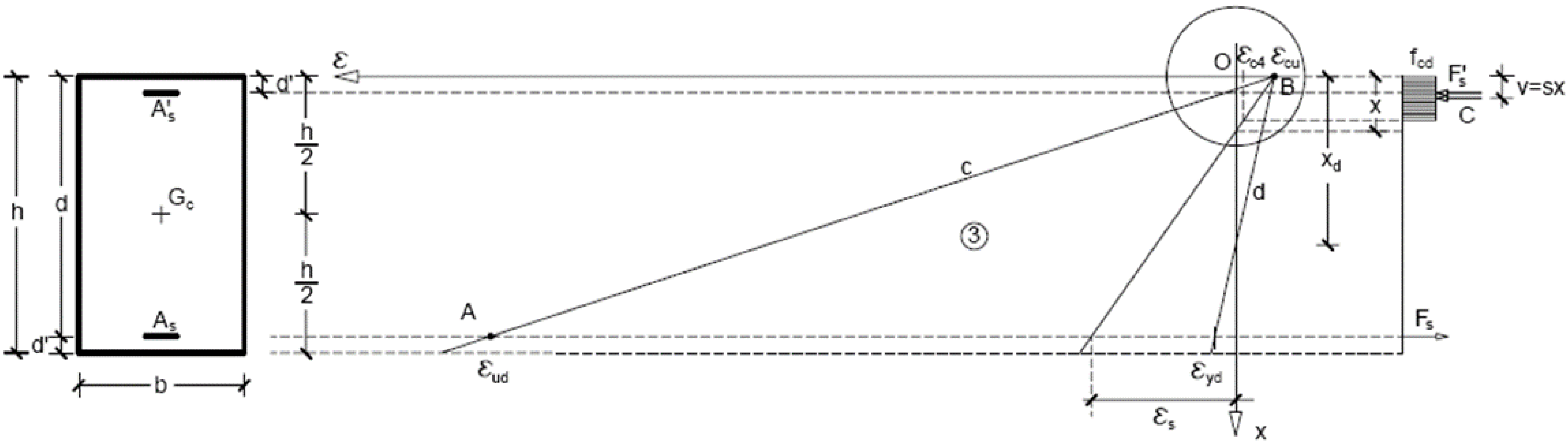
;

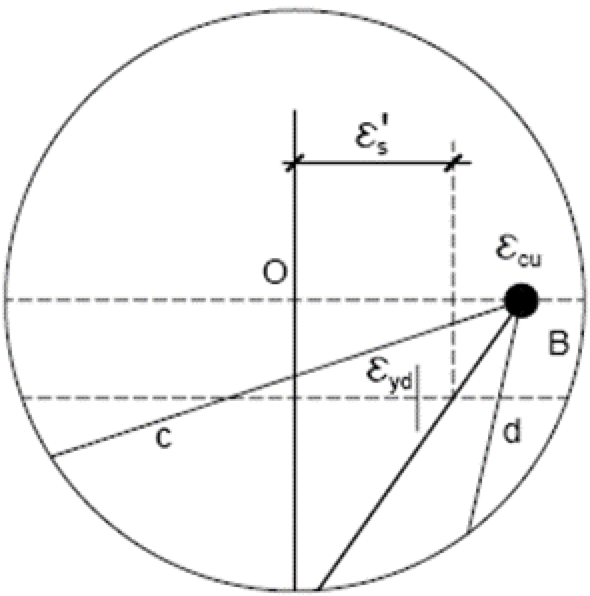
Campo 2 $\left(\xi_b = 0 \le \xi \le \xi_c = 0.049\right)$

$$\begin{cases} N = \alpha(\xi) f_{cd}\, b\, \xi\, d - \sigma_s A_s - \sigma'_s A'_s \\ M = \alpha(\xi) f_{cd}\, b\, \xi\, d \left[\dfrac{h}{2} - s(\xi)\xi d\right] + \left(\sigma_s A_s - \sigma'_s A'_s\right)\dfrac{d - d'}{2} \end{cases}$$



Campo 3 $(\xi_c = 0.049 \le \xi \le \xi_d = 0.641)$

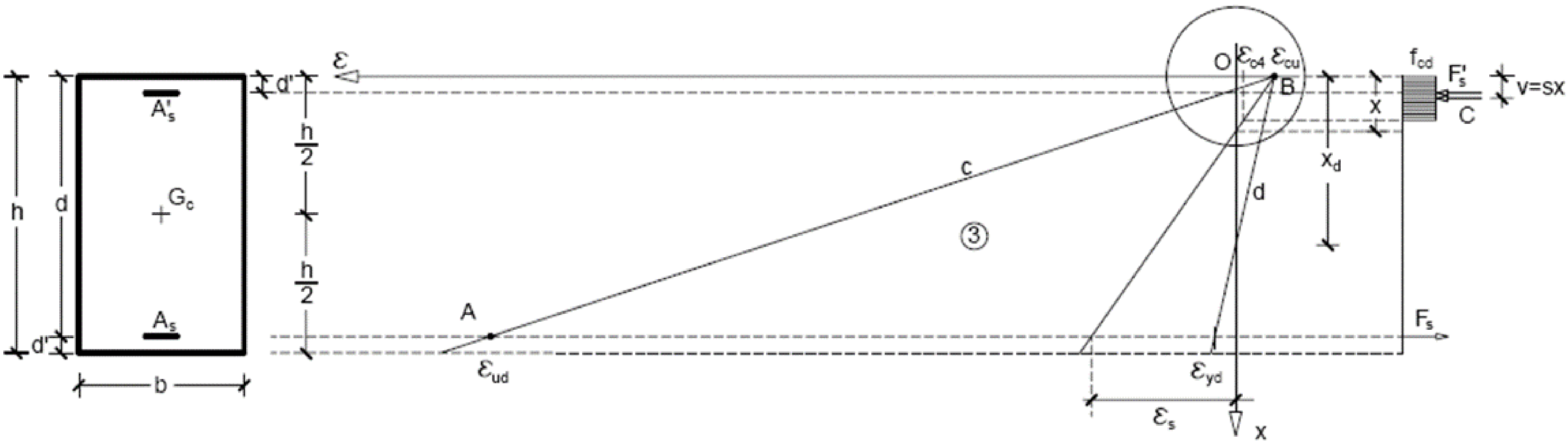




$$\varepsilon'_s = \frac{x - d'}{x}\varepsilon_{cu}$$

$$\Rightarrow \begin{cases} \sigma'_s = -E_s \varepsilon'_s & \text{se} \quad |\varepsilon'_s| \le \varepsilon_{yd} \\ \sigma'_s = -\left[(k-1)\dfrac{|\varepsilon'_s| - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1\right] f_{yd} & \text{se} \quad |\varepsilon'_s| > \varepsilon_{yd} \end{cases}$$

Campo 3 $\left(\xi_c = 0.049 \leq \xi \leq \xi_d = 0.641\right)$

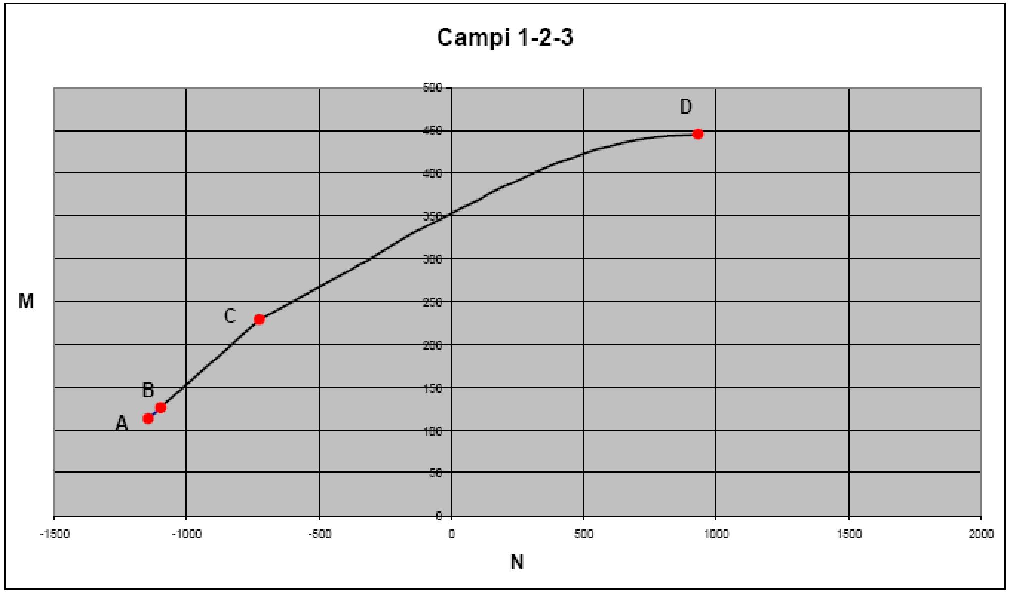


$$\varepsilon_s = \frac{d - x}{x}\,\varepsilon_{cu} \quad \Rightarrow \quad \sigma_s = \left[(k-1)\,\frac{\varepsilon_s - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1\right] f_{yd}$$

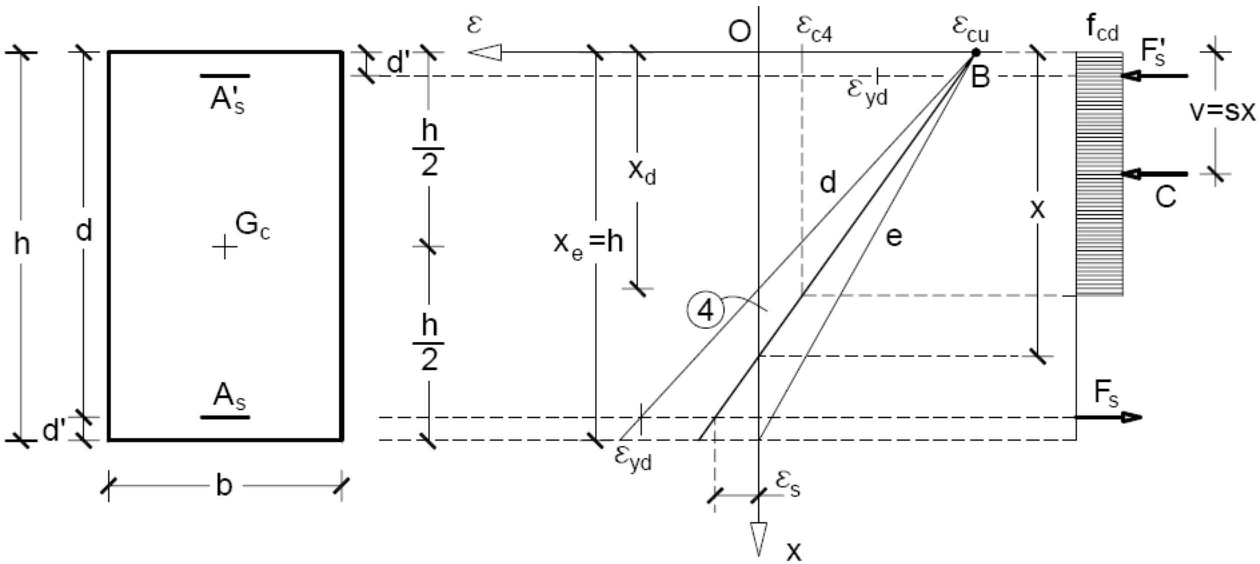
Campo 3 $(\xi_c = 0.049 \leq \xi \leq \xi_d = 0.641)$

$$\begin{cases} N = 0.8 f_{cd} b \xi d - \sigma_s A_s - \sigma'_s A'_s \\ M = 0.8 f_{cd} b \xi d \left[\dfrac{h}{2} - 0.4 \xi d \right] + (\sigma_s A_s - \sigma'_s A'_s) \dfrac{d - d'}{2} \end{cases}$$

$$\sigma'_s = \left[(k-1) \frac{\varepsilon'_s - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1 \right] f_{yd}$$



Campo 4 $\left(\xi_d = 0.641 \le \xi \le \xi_e = 1 + \delta\right)$

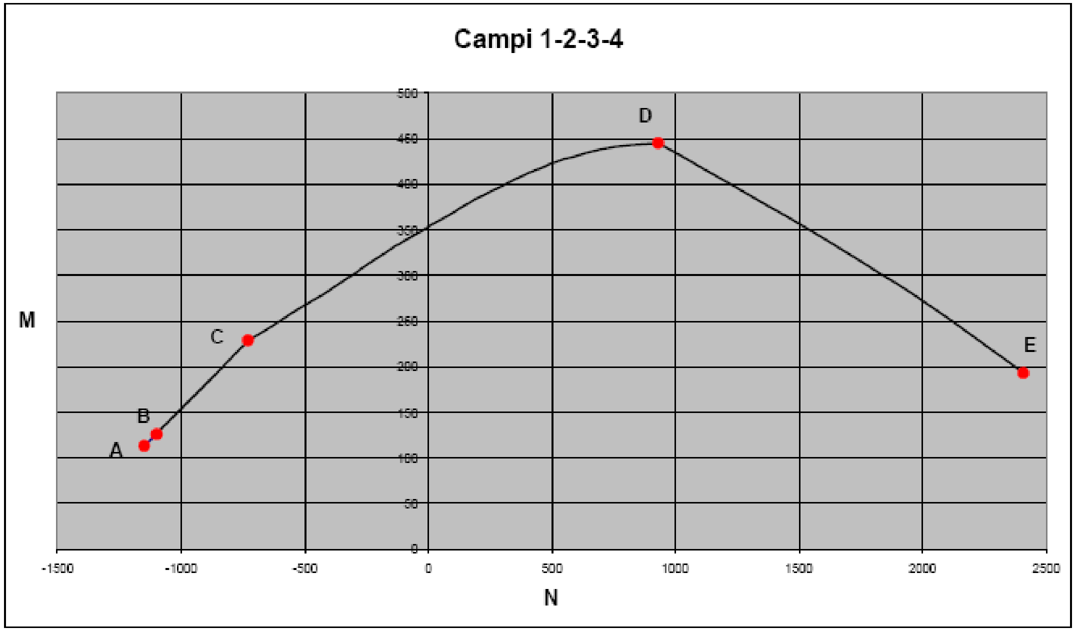


$$\varepsilon_s = \frac{d - x}{x}\varepsilon_{cu} = \frac{1 - \xi}{\xi}\varepsilon_{cu} \qquad \Rightarrow \qquad \sigma_s = E_s \varepsilon_s .$$

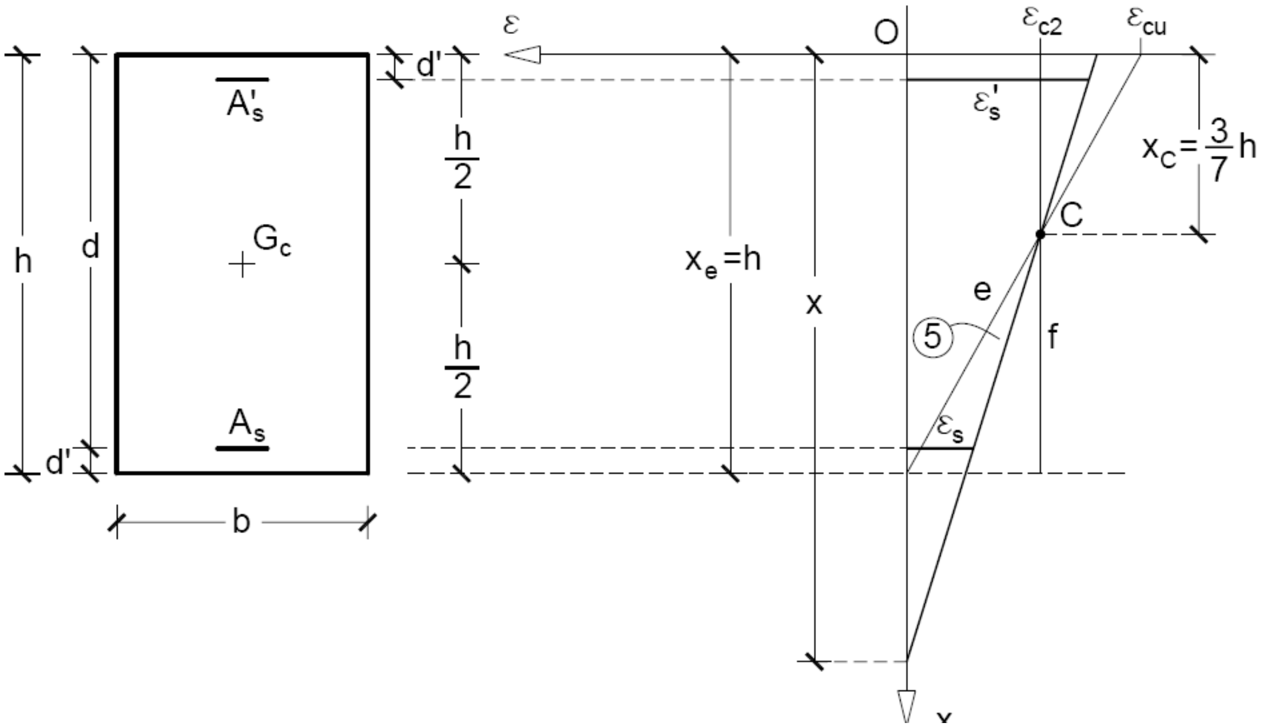
$$\varepsilon'_s = \frac{x - d'}{x}\varepsilon_{cu} = \frac{\xi - \delta}{\xi}\varepsilon_{cu} \qquad \Rightarrow \qquad \sigma'_s = -\left[(k-1)\frac{\varepsilon'_s - \varepsilon_{yd}}{\varepsilon_{ud} - \varepsilon_{yd}} + 1\right] f_{yd} .$$

Campo 4 $\left(\xi_d = 0.641 \le \xi \le \xi_e = 1 + \delta\right)$

$$\begin{cases} N_e = 0.8 f_{cd} bh - \sigma_s A_s - \sigma'_s A'_s \\ M_e = 0.8 f_{cd} bh \left[\dfrac{h}{2} - 0.4h\right] + \left(\sigma_s A_s - \sigma'_s A'_s\right)\dfrac{d - d'}{2} = 0.08 f_{cd} bh^2 + \left(\sigma_s A_s - \sigma'_s A'_s\right)\dfrac{d - d'}{2} \end{cases}$$



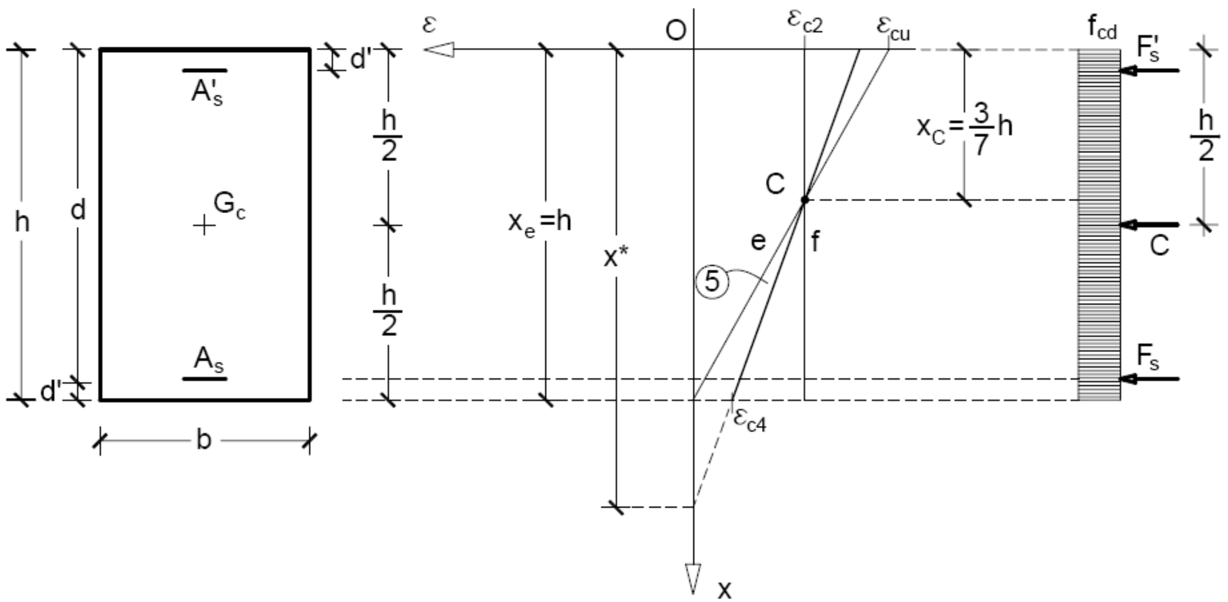
Campo 5 $\left(\xi_e = 1+\delta \le \xi < \xi_f = \infty\right)$



$$\frac{\varepsilon_s}{x-d} = \frac{\varepsilon_{c2}}{x-\frac{3}{7}h} \Rightarrow \varepsilon_s = \frac{x-d}{x-\frac{3}{7}h}\varepsilon_{c2} \Rightarrow \begin{cases} \sigma_s = -E_s\varepsilon_s & \text{se} \quad \varepsilon_s \le \varepsilon_{yd} \\ \sigma_s = -\left[(k-1)\frac{\varepsilon_s - \varepsilon_{yd}}{\varepsilon_{ud}-\varepsilon_{yd}} + 1\right]f_{yd} & \text{se} \quad \varepsilon_s > \varepsilon_{yd} \end{cases}$$

$$\frac{\varepsilon'_s}{x-d'} = \frac{\varepsilon_{c2}}{x-\frac{3}{7}h} \Rightarrow \varepsilon'_s = \frac{x-d'}{x-\frac{3}{7}h}\varepsilon_{c2} \Rightarrow \sigma'_s = -\left[(k-1)\frac{\varepsilon'_s - \varepsilon_{yd}}{\varepsilon_{ud}-\varepsilon_{yd}} + 1\right]f_{yd} \; .$$

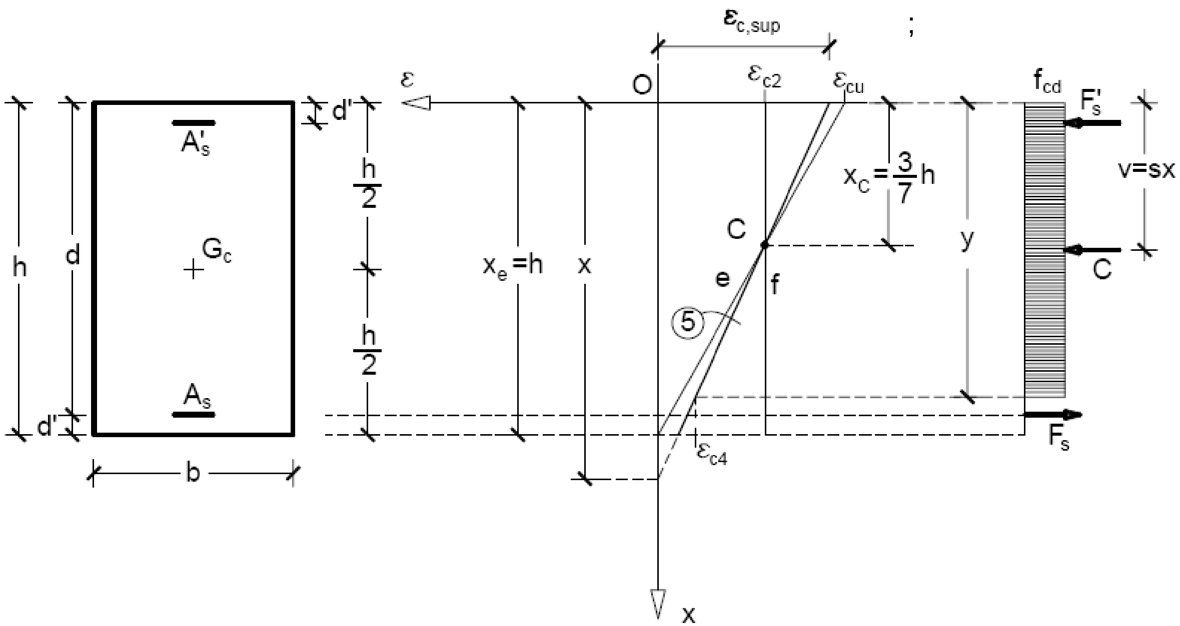
Campo 5 $\left(\xi_e = 1 + \delta \leq \xi < \xi_f = \infty\right)$



$$\frac{\varepsilon_{c2}}{x^* - \frac{3}{7}h} = \frac{\varepsilon_{c4}}{x^* - h} \qquad \Rightarrow \qquad x^* = \frac{\varepsilon_{c2} - \frac{3}{7}\varepsilon_{c4}}{\varepsilon_{c2} - \varepsilon_{c4}} h = 1.307692 \cdot h$$

Stati Limite Ultimi per Tensioni normali

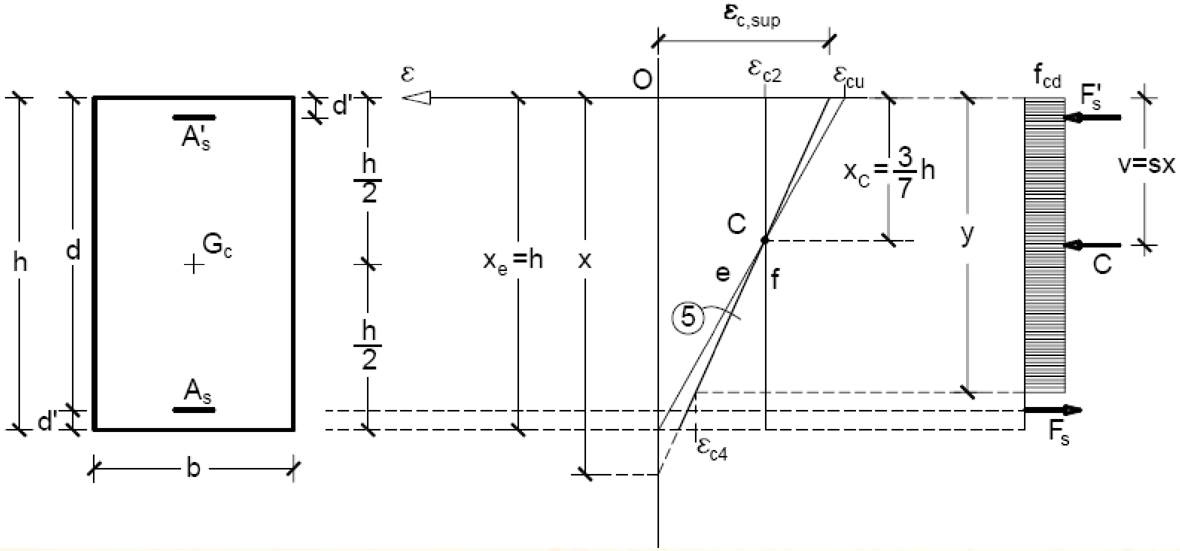
Domini di interazione



$$h \leq x < x^*$$

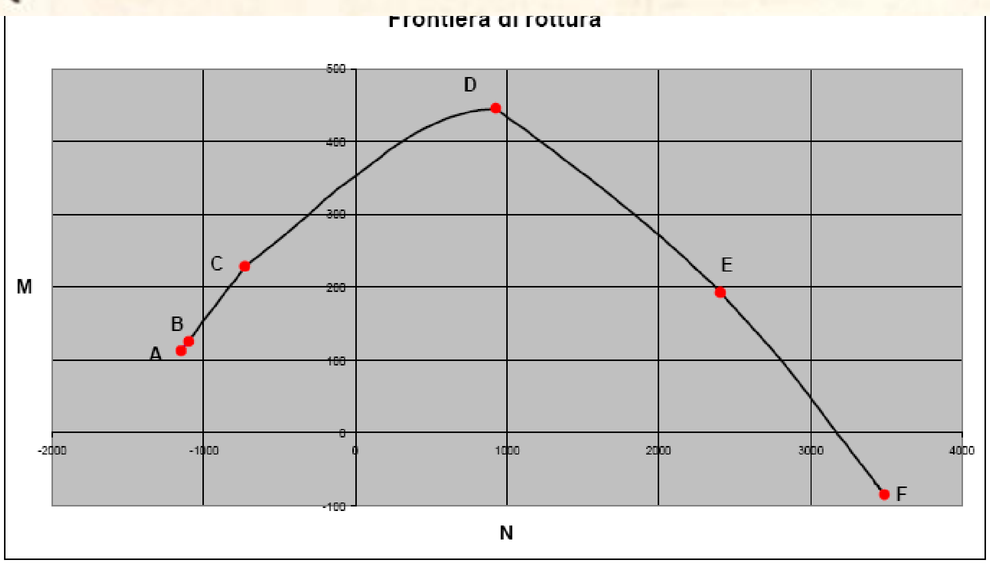
$$\alpha(x) = \frac{y}{x} = 0.65 + 0.15\frac{h}{x}$$

x/h	y/h	α
1.00	0.8000	0.800
1.05	0.8325	0.793
1.10	0.8650	0.786
1.15	0.8975	0.780
1.20	0.9300	0.775
1.25	0.9625	0.770
1.307692	1.0000	0.765



$$h \le x < x^*$$

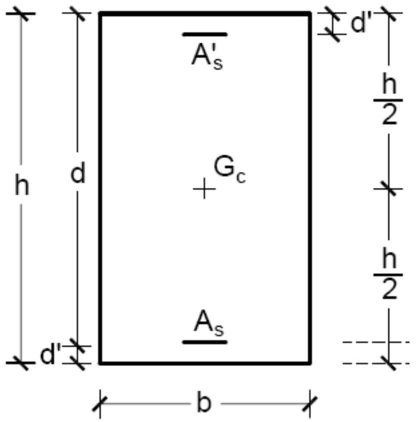
$$\begin{cases} N = \alpha(x) f_{cd} bx - \sigma_s A_s - \sigma'_s A'_s \\ M = \alpha(x) f_{cd} bx \left[\frac{h}{2} - s(x)x \right] + (\sigma_s A_s - \sigma'_s A'_s) \frac{d-d'}{2} \end{cases} \quad \text{per} \quad h \le x \le x^* = 1.307692h$$

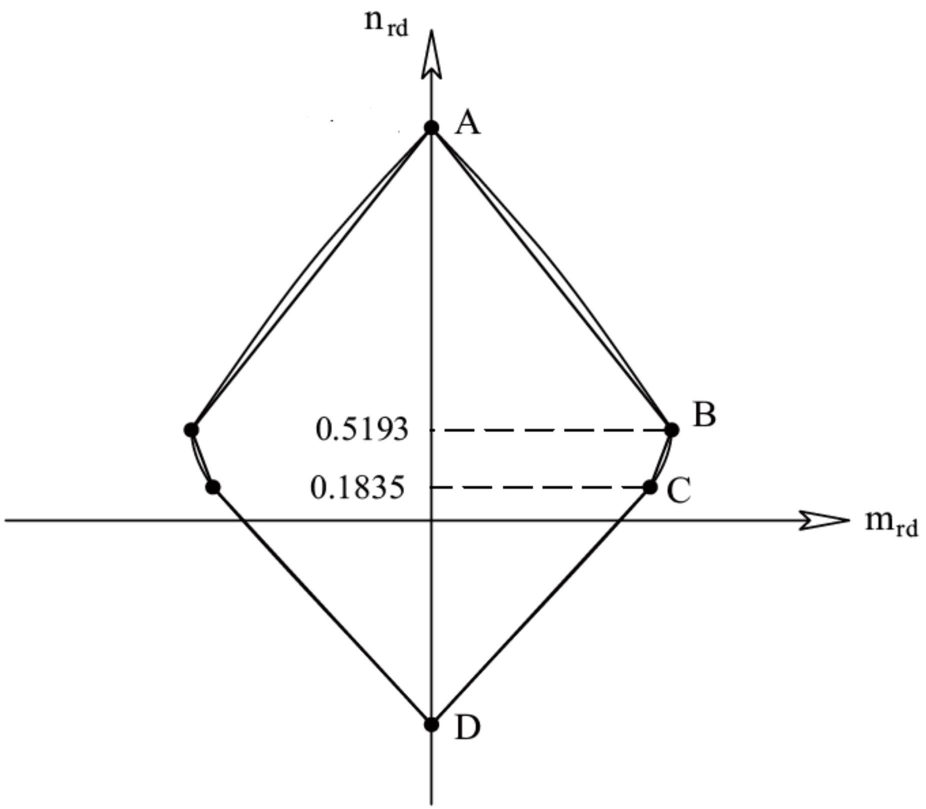


$$\begin{cases} N = f_{cd} bh - \sigma_s A_s - \sigma'_s A'_s \\ M = (\sigma_s A_s - \sigma'_s A'_s) \frac{d-d'}{2} \end{cases} \quad \text{per} \quad x > x^*$$

Domini di interazione semplificati

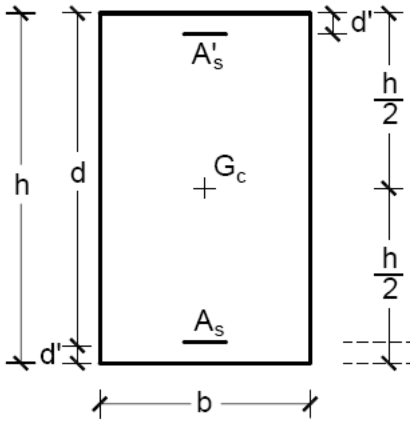
Essendo il dominio convesso è possibile adottare domini semplificati definiti considerando solo alcune rette di rottura

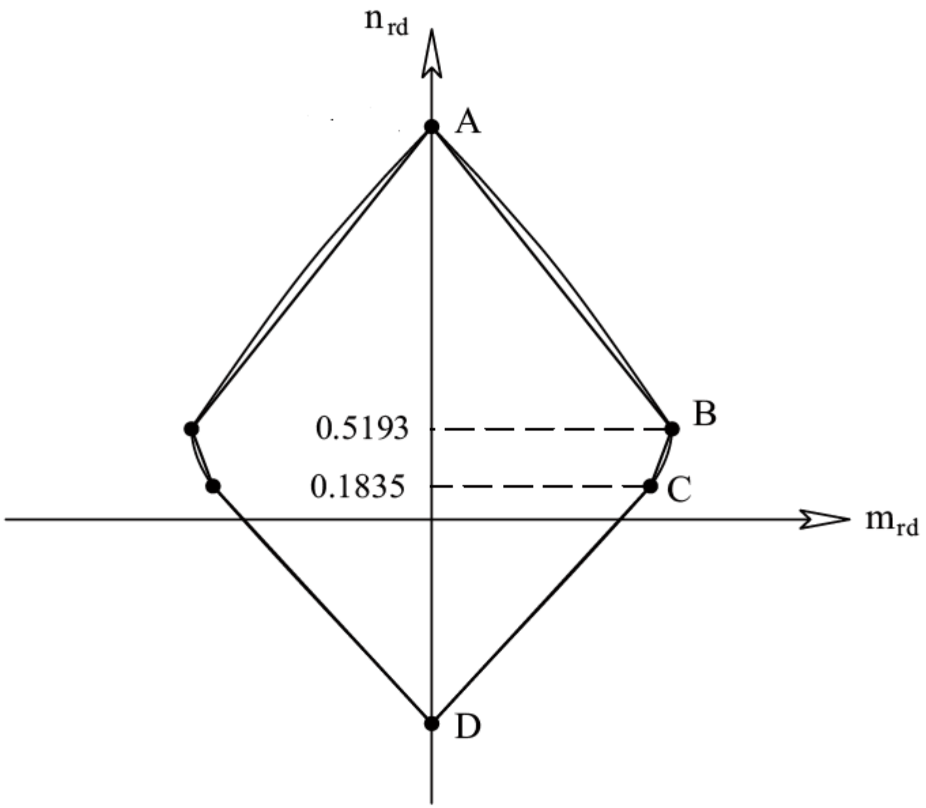




Se $A'_s = A_s$

Domini di interazione semplificati





Trazione pura

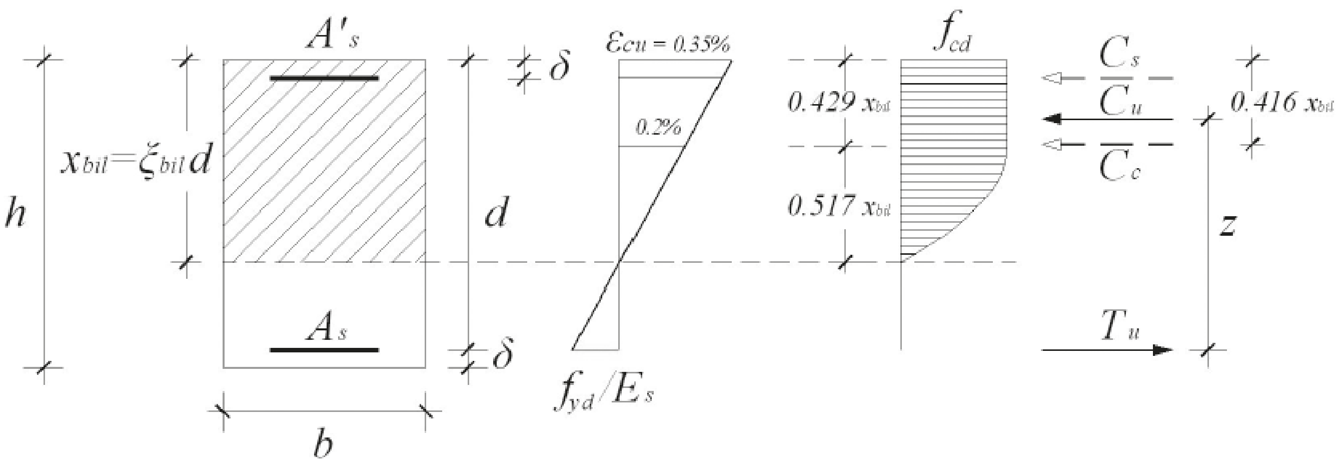
$$\begin{cases} N_D = -2kf_{yd}A_s \\ M_D = 0 \end{cases}$$

Compressione pura

$$\begin{cases} N_A = f_{cd}b(d+\delta) + 2E_s\sigma_s A_s \\ M_A = 0 \end{cases}$$

Stati Limite Ultimi per Tensioni normali

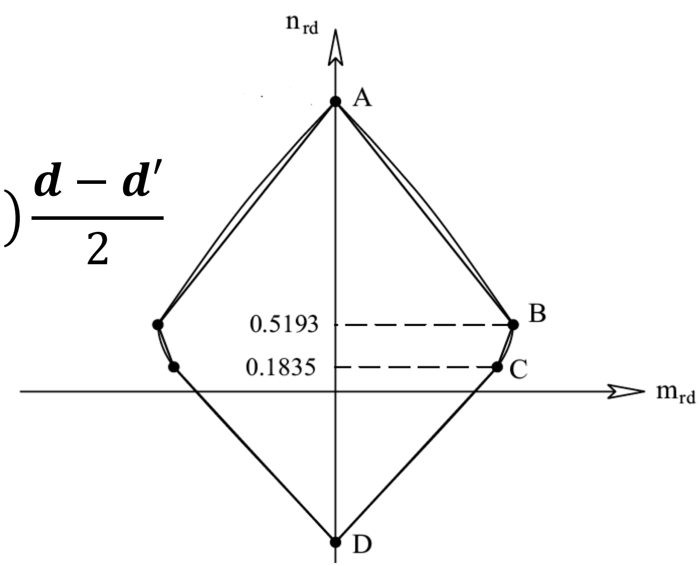
Domini di interazione semplificati



Rottura bilanciata B

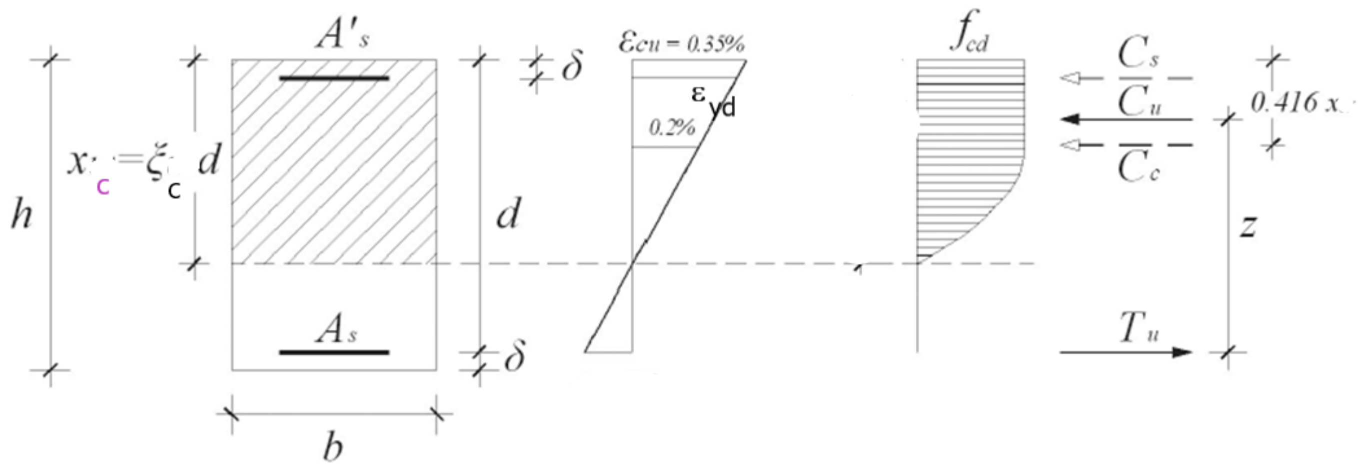
$$\boldsymbol{N_B} = 0.5128\boldsymbol{f_{cd}}bd - \boldsymbol{f_{yd}A_s} + \sigma'_s A'_s$$

$$\boldsymbol{M_B} = 0.5128\boldsymbol{f_{cd}}bd\left[\frac{\boldsymbol{h}}{2} - 0.2564\boldsymbol{d}\right] + \left(\boldsymbol{f_{yd}A_s} + \boldsymbol{\sigma'_s A'_s}\right)\frac{\boldsymbol{d - d'}}{2}$$

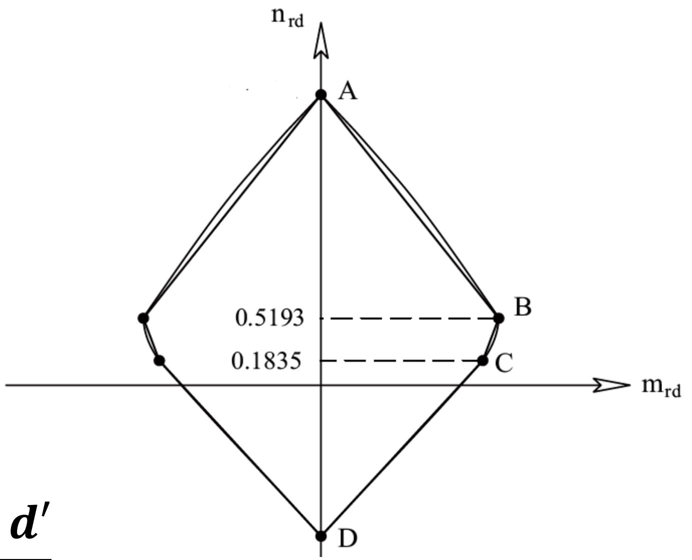


Stati Limite Ultimi per Tensioni normali

Domini di interazione semplificati



Rottura con armatura compressa al limite elastico C



$$\xi_C = \lambda \frac{\varepsilon_{cu}}{\varepsilon_{cu} - \varepsilon_{yd}} = \lambda \frac{3.5}{3.5 - 1.956} = 2.267\,\lambda$$

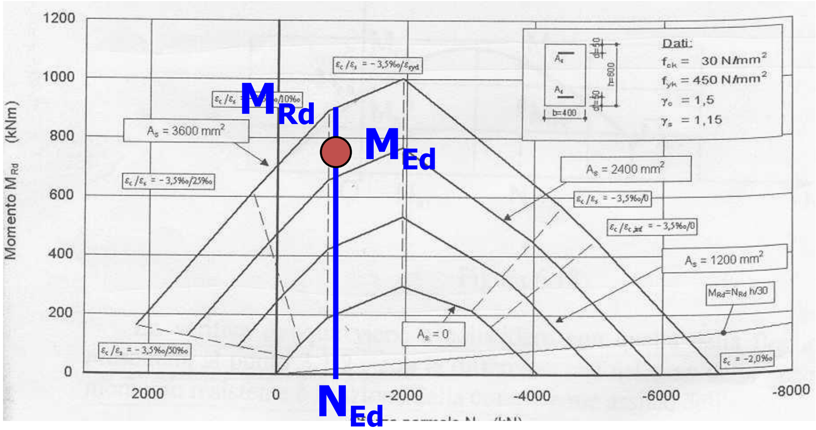
$$\begin{cases} \boldsymbol{N_C} = 1.81 \boldsymbol{f_{cd}} b\lambda d + \boldsymbol{f_{yd}} \boldsymbol{A'_s} - \boldsymbol{\sigma_s} \boldsymbol{A_s} \\ \boldsymbol{M_C} = 1.81 \boldsymbol{f_{cd}} b\lambda d \left[\dfrac{\boldsymbol{h}}{2} - 0.91 \boldsymbol{\lambda d}\right] + (\boldsymbol{f_{yd}} \boldsymbol{A'_s} + \boldsymbol{\sigma_s} \boldsymbol{A_s}) \dfrac{\boldsymbol{d} - \boldsymbol{d'}}{2} \end{cases}$$

Stati Limite Ultimi per Tensioni normali

Flessione composta retta: Verifica

La verifica è soddisfatta se risulta:

$$M_{Ed} \leq M_{Rd}\,(N_{Ed})$$



- **Equazione di equilibrio alla traslazione**

$$F'_s + F_c - F_s = N_{sd} \quad \Rightarrow \quad \xi$$

- **Equilibrio alla rotazione intorno al baricentro**

$$M_{Rd} = 0.8\, f_{cd}\, b\, \xi\, d\, [(h-0.4\, d)/2] + f_{yd}\, A_s'\, [(h/2) - d'] + f_{yd}\, A_s\, [(h/2) - d']$$

Flessione composta retta: Verifica

Nel caso di sezione caricate assialmente è necessario considerare una eccentricità del carico *e* :

$$e \geq \begin{cases} h/200 \\ 20 \text{ mm} \end{cases}$$

essendo *h* l'altezza libera di inflessione del pilastro

Questa verifica si considera implicitamente soddisfatta se si assume:

$$N_{Rd} = 0.8 f_{cd} A_c + A_{s,tot} f_{yd}$$

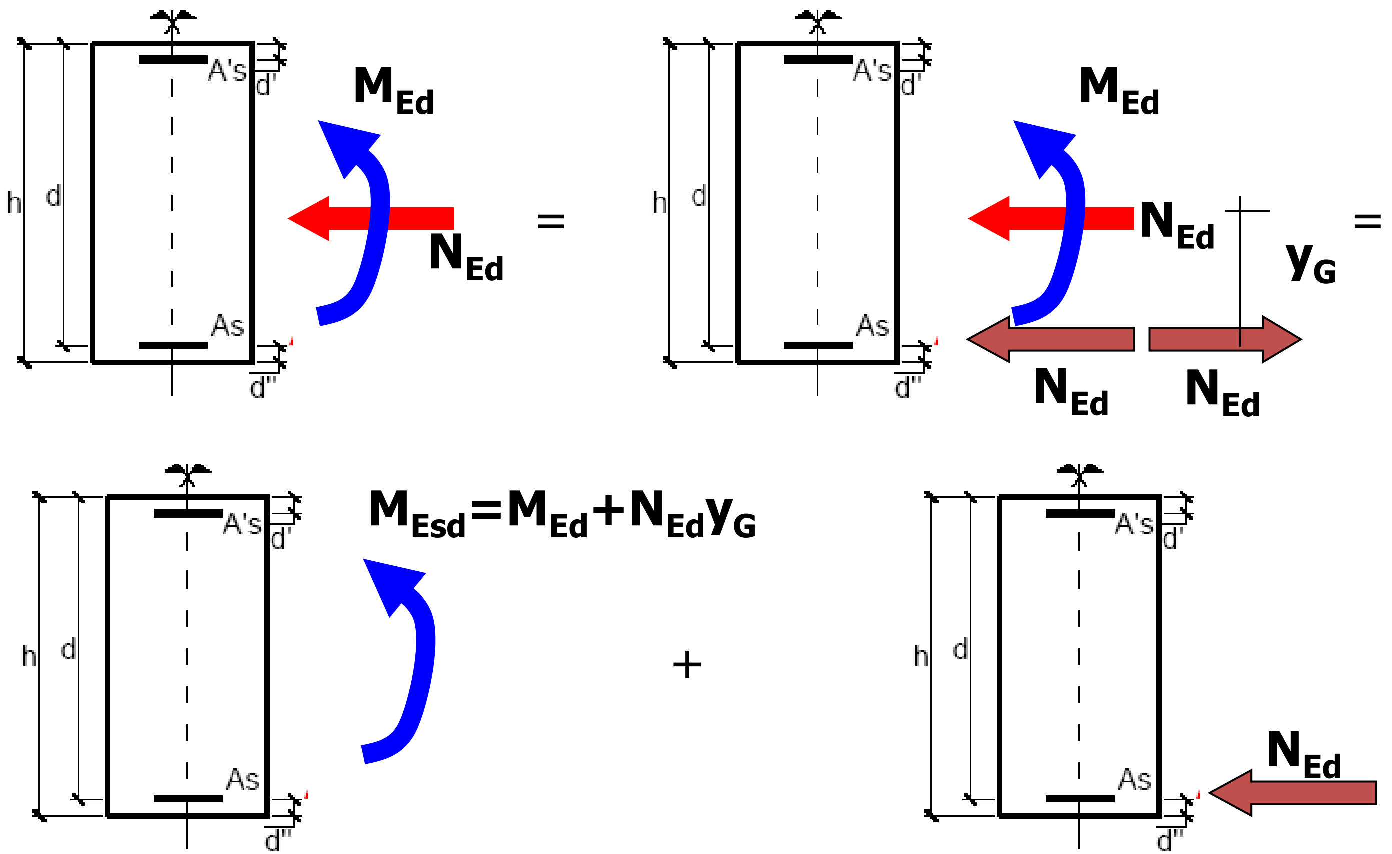
Le incognite sono:

- **le dimensioni geometriche della sezione: b ed h;**
- **l'area di acciaio: A_s , A_s';**
- **la retta di rottura.**

Si fissano x per duttilità, μ, e una delle dimensioni sulla base di esigenze architettoniche

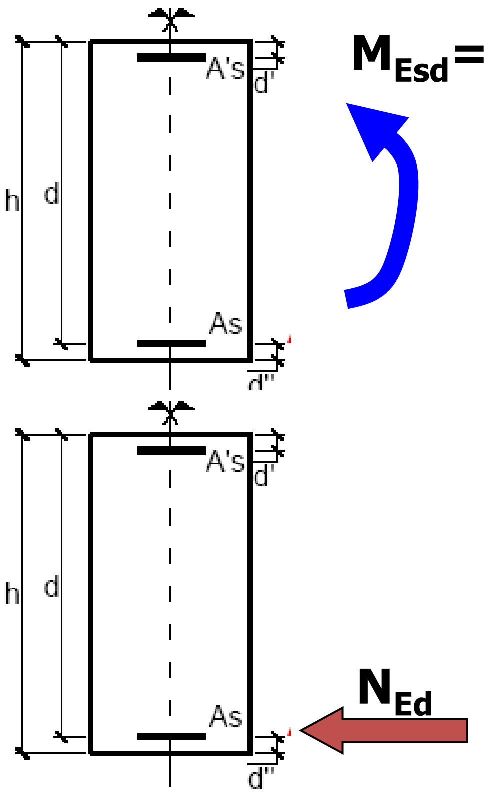
$$\begin{cases} \boldsymbol{N_{sd}} = \alpha \boldsymbol{f_{cd}} b \xi d - \sigma_s \boldsymbol{A_s} + \sigma'_s A'_s \\ \boldsymbol{M}_{sd} = \alpha \boldsymbol{f_{cd}} bd \left[\frac{\boldsymbol{h}}{2} - \boldsymbol{s\xi d} \right] + (\boldsymbol{\sigma_s A_s} + \boldsymbol{\sigma'_s A'_s}) \frac{\boldsymbol{d} - \boldsymbol{d'}}{2} \end{cases}$$

Sezione assegnata



Stati Limite Ultimi per Tensioni normali

Flessione composta retta: Progetto



$M_{Esd}=M_{Ed}-N_{Ed}y_G$

Progetto dell'armatura in zona tesa e compressa per sola flessione retta

A_{s1} ; A_{s1}'

N_{Ed}

Progetto dell'armatura per sforzo di compressione agente in corrispondenza della sola armatura in zona tesa

$A_{s2} = N_{Ed}/f_{yd}$

Armatura finale: $A_s=A_{s1}-A_{s2}$ $A_s'=A_{s1}'$